St. John the Baptist, Findon

Policy for written calculations - Mathematics

Date: May 2023

Document summary

This document will be used from September 2023, when all calculation methods will be taught again. It sets out progression from mental methods through to standard formal written calculations.

Structured and images are used through all stages to ensure that concepts are understood and embedded. The concrete, pictorial and abstract (CPA) approach will be used in every classroom so that children can truly understand what it is they are doing, and why, rather than simply learning the written methods.

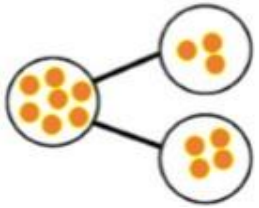
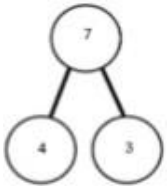
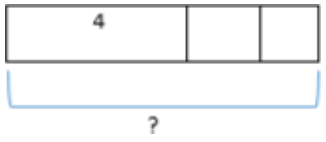
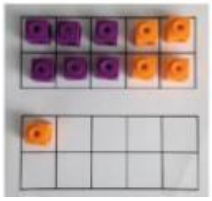
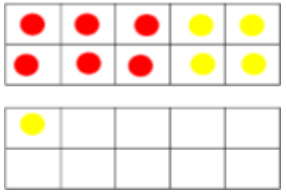
Precise mathematical vocabulary is used by all adults in school to reduce cognitive load and we encourage children to do the same.

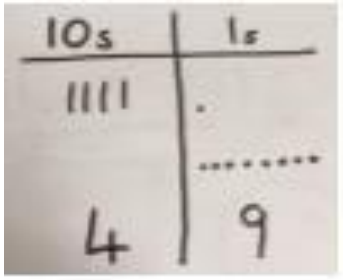
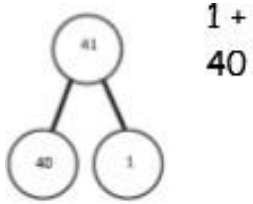
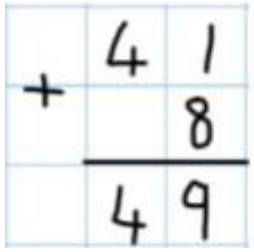
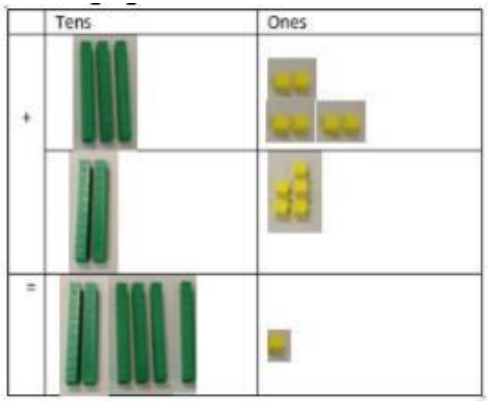
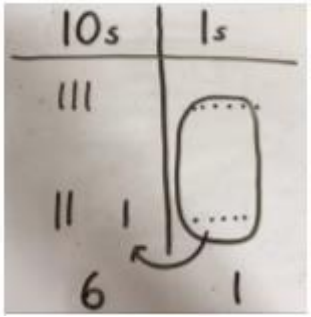
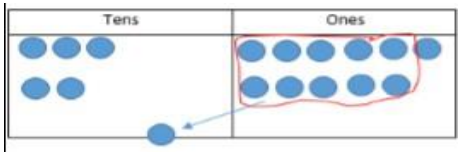
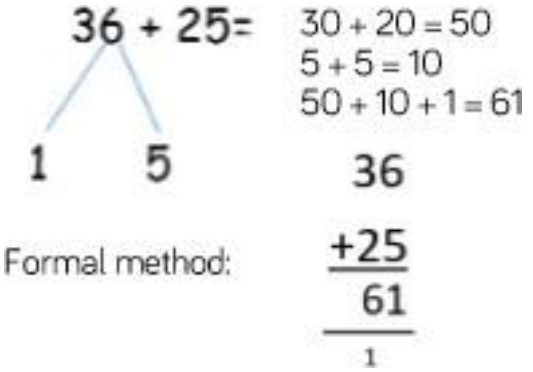
It is important to offer calculations in a range of representations. This includes part, part whole; bar modelling; the tens frame; missing numbers and expressions where the = sign is not always in the traditional place.

	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
Addition	Combining two parts to make a whole: part whole model. Starting at the bigger number and counting on. Regrouping to make 10.	Adding three single digits. Column method – no regrouping.	Column method- regrouping. (up to 3 digits)	Column method- regrouping. (up to 4 digits)	Column method- regrouping. (with more than 4 digits) (Decimals- with the same amount of decimal places)	Column method- regrouping. (Decimals- with different amounts of decimal places)
Subtraction	Taking away ones Counting back Find the difference Part whole model Make 10	Counting back Find the difference Part whole model Make 10 Column method- no regrouping	Column method with regrouping. (up to 3 digits)	Column method with regrouping. (up to 4 digits)	Column method with regrouping. (with more than 4 digits) (Decimals- with the same amount of decimal places)	Column method with regrouping. (Decimals- with different amounts of decimal places)
Multiplication	Doubling Counting in multiples Arrays (with support)	Doubling Counting in multiples Repeated addition Arrays- showing commutative multiplication	Counting in multiples Repeated addition Arrays- showing commutative multiplication Grid method	Column multiplication (2 and 3 digit multiplied by 1 digit)	Column multiplication (up to 4 digit numbers multiplied by 1 or 2 digits)	Column multiplication (multi digit up to 4 digits by a 2 digit number)
Division	Sharing objects into groups Division as grouping	Division as grouping Division within arrays	Division within arrays Division with a remainder Short division (2 digits by 1 digit- concrete and pictorial)	Division within arrays Division with a remainder Short division (up to 3 digits by 1 digit- concrete and pictorial)	Short division (up to 4 digits by a 1 digit number interpret remainders appropriately for the context)	Short division Long division (up to 4 digits by a 2 digit number- interpret remainders as whole numbers, fractions or round)

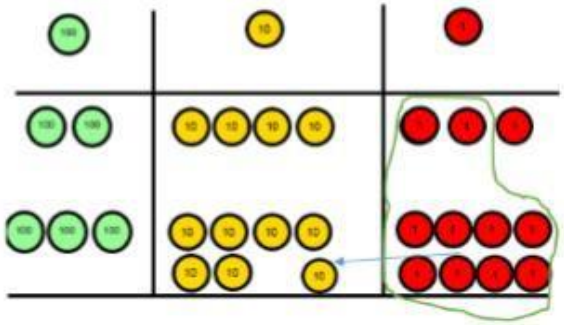
CALCULATION POLICY FOR ADDITION

Key language which should be used: sum, total, parts and wholes, plus, add, altogether, more than, 'is equal to' 'is the same as'.

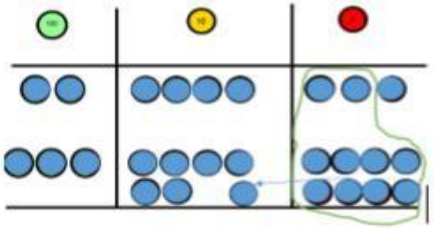
Concrete	Pictorial	Abstract
Combining two parts to make a whole (use other resources too e.g. shells, teddy bears etc.)  	Children to represent the counters using dots. 	**4 + 3 = 7** (four is a part, 3 is a part and the whole is seven.) 
Counting on using number lines by using cubes or Numicon.  	A bar model which encourages the children to count on rather than count all. 	The abstract number line: What is 2 more than 4? What is the sum of 2 and 4? What is the total of 4 and 2? 4+2 
Regrouping to make 10; using ten frames. 6+5  	Children to draw the tens frame. 	Children to develop an understanding of equality. e.g. 6 + □ = 11 6 + 5 = 5 + □ 6 + 5 = □ + 4

TO+O; using base 10 to develop understanding of partitioning and place value. **41+8 =** 	Children to represent the base 10. e.g. lines for tens and dots for ones. 	**= 41+8**  1 + 8 = 9 40 + 9 = 49 
TO+TO; using base 10 to develop understanding of partitioning and place value. **36+25** 	Children to represent the base 10 in a place value chart.  	Looking for ways to make 10. 

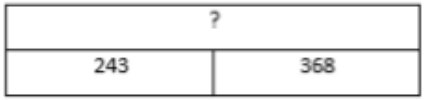
Use of place value counters to add HTO+TO, HTO+HTO etc. When there are 10 ones in the 1s column- we exchange for 1 ten; when there is 10 tens in the 10s column – we exchange for 1 hundred.



Children to represent the counters in a place value chart, circling when they make an exchange.

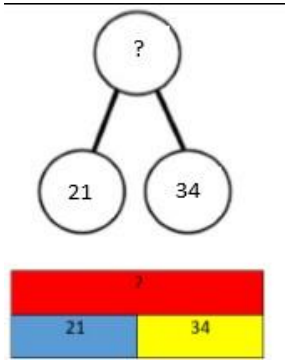


If the children are completing a word problem, draw a bar model to represent what it's asking them to do.



$$\begin{array}{r} 243 \\ +368 \\ \hline 611 \\ \hline 1\ 1 \end{array}$$

Conceptual Variation, different ways to ask children to solve 21+34



Word problems: In year 3, there are 21 children and in year 4, there are 34 children. How many children in total?

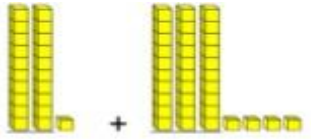
21 + 34 = 55. Prove it

$$\begin{array}{r} 21 \\ +34 \\ \hline \\ \hline \end{array}$$

21 + 34 = ☐

☐ = 21 + 34

What's the sum of twenty one and thirty four?



Always use missing digit problems too:

Tens	Ones
●●	●
●●●	?
?	4

MENTAL CALCULATIONS FOR ADDITION

These are a **selection** of mental calculation strategies:
It is very important that children know and can use their number facts.

Mental recall of number bonds
6 + 4 = 10 □ + 3 = 10 25 + 75 = 100 19 + □ = 20

Use near doubles
6 + 7 = double 6 + 1 = 13

Addition using partitioning and recombining
34 + 45 = (30 + 40) + (4 + 5) = 79

Counting on or back in repeated steps of 1, 10, 100, 1000
86 + 57 = 143 (by counting on in tens and then in ones) 460 - 300 = 160 (by counting back in hundreds)

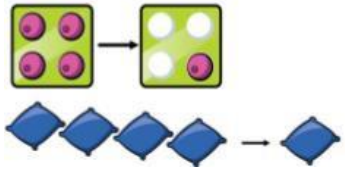
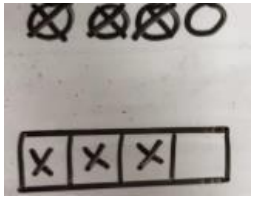
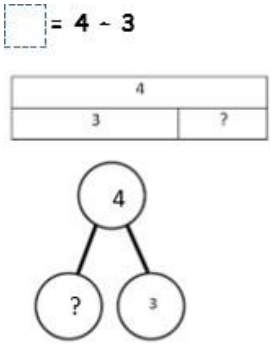
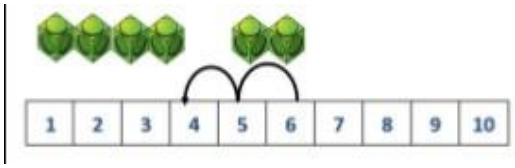
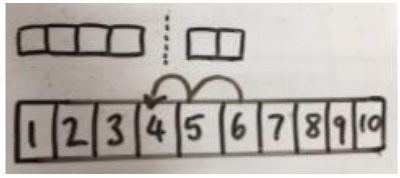
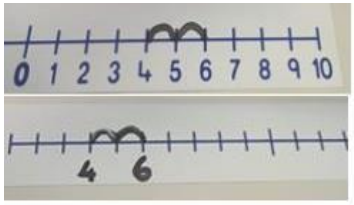
Add the nearest multiple of 10, 100 and 1000 and adjust
24 + 19 = 24 + 20 – 1 = 43 458 + 71 = 458 + 70 + 1 = 529

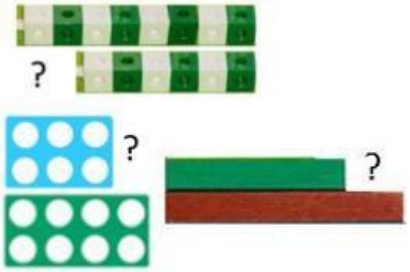
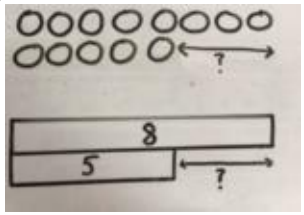
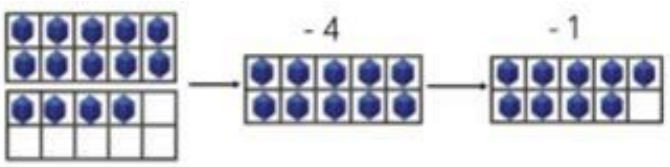
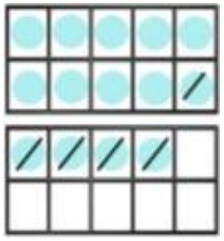
Use the relationship between addition and subtraction
36 + 19 = 55 19 + 36 = 55
55 – 19 = 36 55 – 36 = 19

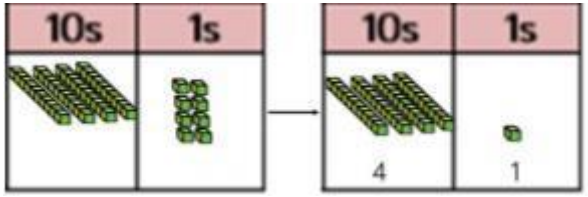
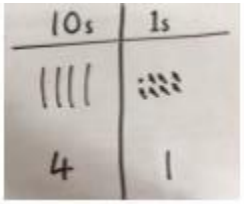
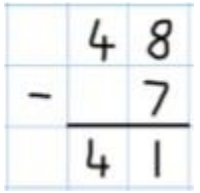
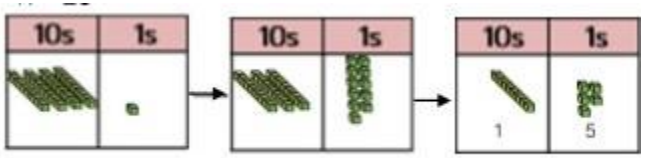
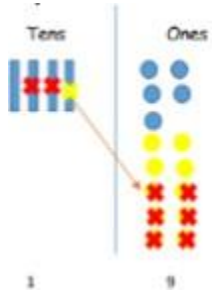
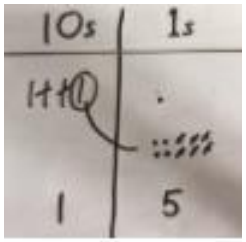
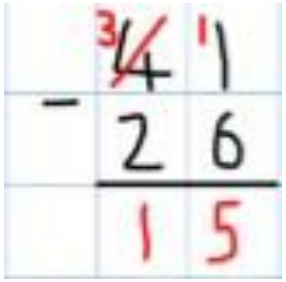
MANY MENTAL CALCULATION STRATEGIES WILL CONTINUE TO BE USED. THEY ARE NOT REPLACED BY WRITTEN METHODS.

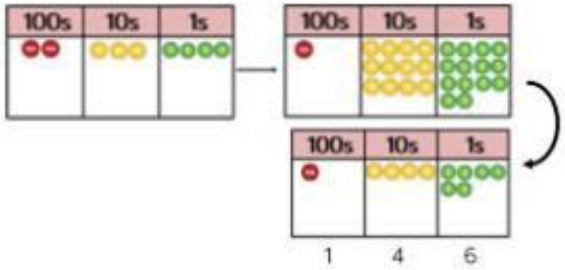
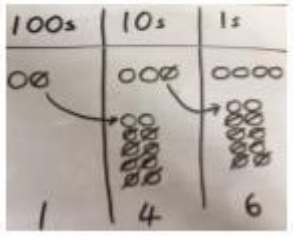
CALCULATION POLICY FOR SUBTRACTION

Key language which should be used: take away, less than, the difference, subtract, minus, fewer, decrease.

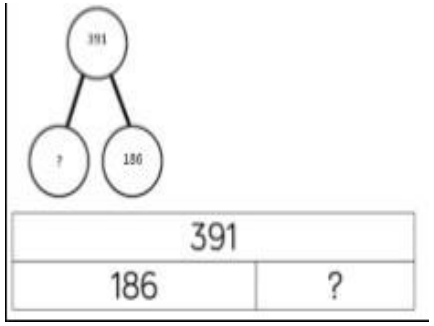
Concrete	Pictorial	Abstract
Physically taking away and removing objects from a whole. **4 -3 = 1** 	Children to draw the concrete resources they are using and cross out. The bar model can also be used. 	**4 + 3 = 1** 
Counting back (using number line or number tracks) Children to start with 6 and count back 2. 	Children to represent what they see pictorially. 	Children to represent the calculation on a number line or number track and show their jumps. Encourage children to use an empty number line. 

Finding the difference;  	Children to draw the counters/other concrete objects which they have used. The children should be encouraged to use the bar model.  	Find the difference between 8 and 5. 8 – 5, the difference is ☐ Children to 14 - 5 = 9 9 - 6 = 8 – ve the same difference. 4 1
Making 10; using the tens frames. **14-5** =  	Children to present the tens frame pictorially and discuss what they did to make 10. 	14 – 5 = 9 You also want children to see related facts e.g. 15 – 9 = 5 Children to represent how they have solved it e.g. 14 - 5 = 9 14 is made up of 5, 5 and 4 so I can subtract one 5 to be left with 4 and 5 5 5 14 - 5 = 9 5 is made up of 4 and 1 so I can subtract 4 to make 10 and then 1 to get to 9 4 1 Children to show how they can make 10 by partitioning.

Column method; using base 10. **48-7** 	Children to represent the base 10 pictorially. 	Column method or children could count back 7. 
Column method; using base 10 and having to exchange. **45-26**  1) Start by partitioning 45 2) Exchange one ten for ten more ones 3) Subtract the ones, then the tens. **41-26** 	Children to represent the base 10 pictorially, remembering to show the exchange.  	Formal column method. Children must understand that when they have exchanged the 10 they still have: 45 =30 +15 41 = 30 +11. 

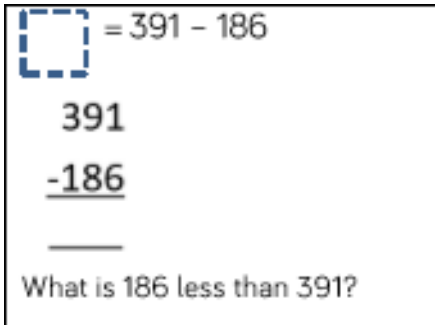
Column method; using place value counters. **234-88** 	Represent the place value counters pictorially; remembering to show what has been exchanged. 	Formal column method. Children must understand what has happened when they have crossed out digits. $\begin{array}{r} {}^{2}2{}^{1}\cancel{3}4 \\ -\ 88 \\ \hline 6 \end{array}$

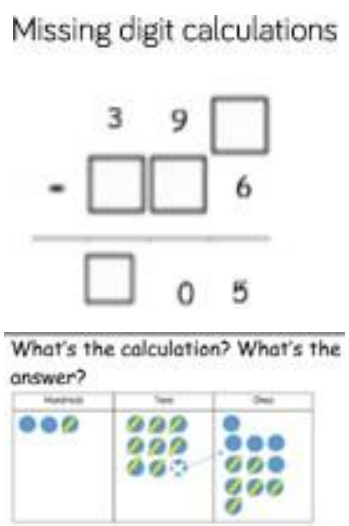
Conceptual Variation, different ways to ask children to solve 391-186



Raj spent £391, Timmy spent £186. How much more did Raj spend?

Calculate the difference between 391 and 186.





MENTAL CALCULATIONS FOR SUBTRACTION

These are a **selection** of mental calculation strategies:
It is important that children know and can apply their number facts.

Mental recall of addition and subtraction facts

10 – 6 = 4 17 - □ = 11 20 - 17 = 3 10 - □ = 2

Find a small difference by counting up

82 – 79 = 3

Counting on or back in repeated steps of 1, 10, 100, 1000

86 - 52 = 34 (by counting back in tens and then in ones) 460 - 300 = 160 (by counting back in hundreds)

Subtract the nearest multiple of 10, 100 and 1000 and adjust

24 - 19 = 24 - 20 + 1 = 5 458 - 71 = 458 - 70 - 1 = 387

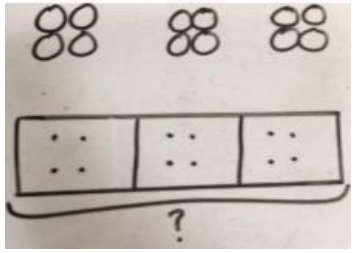
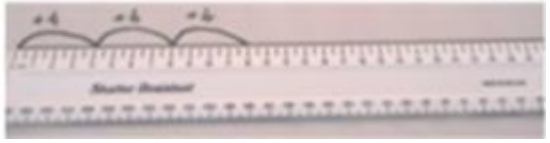
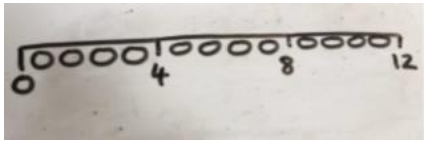
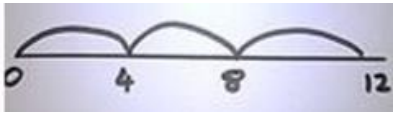
Use the relationship between addition and subtraction

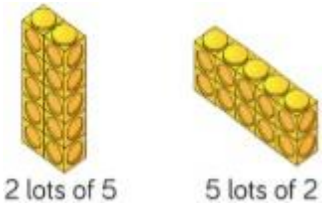
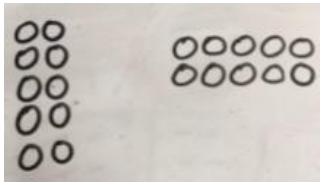
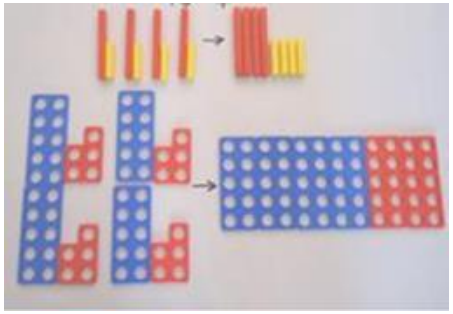
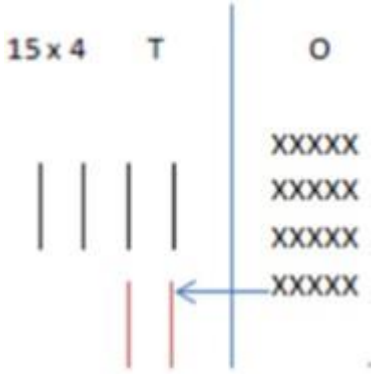
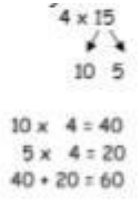
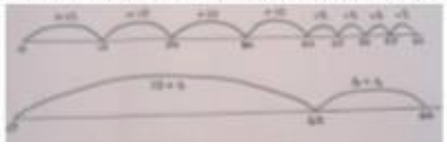
36 + 19 = 55 19 + 36 = 55 55 – 19 = 36 55 – 36 = 19

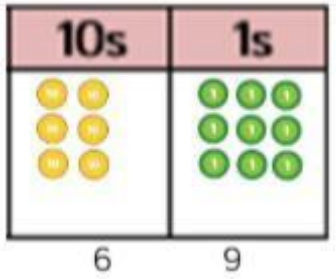
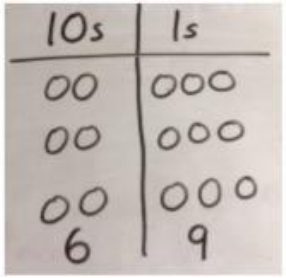
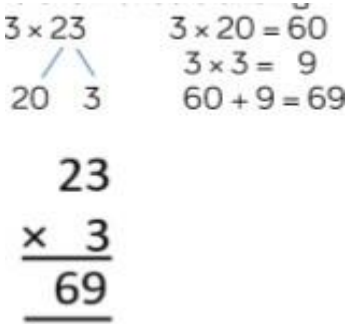
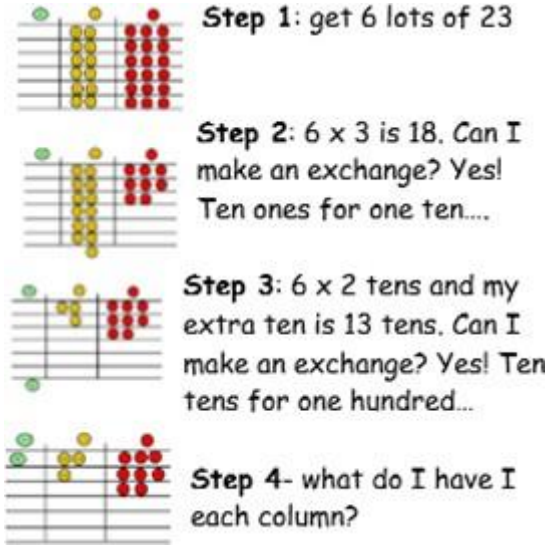
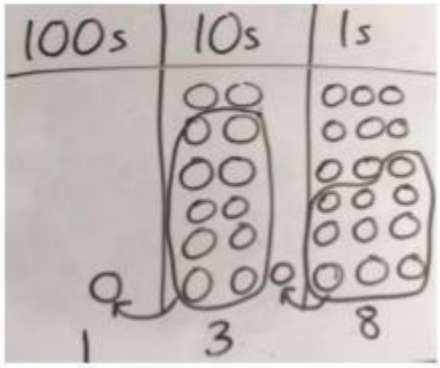
MANY MENTAL CALCULATION STRATEGIES WILL CONTINUE TO BE USED. THEY ARE NOT REPLACED BY WRITTEN METHODS.

CALCULATION POLICY FOR MULTIPLICATION

Key language which should be used: double times, multiplied by, the product of, groups of, lots of.

Concrete	Pictorial	Abstract
Repeated grouping/repeated addition (does not have to be restricted to cubes) 3 x 4 There are 3 equal groups, with 4 in each group. 4+4+4 	Children to represent the practical resources in a picture and use a bar model. 	**3 x 4 = 12** **12= 3 x 4** **4+4+4 = 12**
Number Lines to show repeated groups. 3 x 4   Cuisenaire rods can be used too.	Represent this pictorially alongside a number line e.g. 	Abstract number line showing three jumps of four. 3 x 4 = 12 

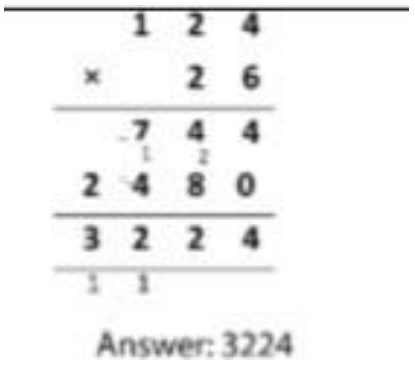
Use arrays to illustrate commutativity; (counters and other objects can also be used) 2 x 5 = 5 x 2  	Children to represent the arrays pictorially. 	Children to be able to use an array to write a range of calculations e.g. 10 = 2 x 5 5 x 2 = 10 2 + 2 + 2 + 2 + 2 = 10 10 = 5 + 5
Partition to multiply; using Numicon, base 10 or Cuisenaire rods. **15 x 4** 	Children to represent the concrete manipulatives pictorially.  	Children to be encouraged to show the steps they have taken.  A number line can also be used 

Formal Column Method; with place value counters (base ten can also be used) **3 x 23** 	Children to represent the counters pictorially. 	Children to record what it is they are doing to show understanding. 
Formal Column Method with place value counters. 6 x 23 	Children to represent the counters/ base 10 pictorially. 	6 x 23 = 23 x 6 138 1 1

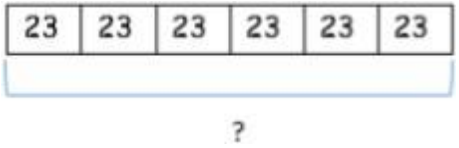
When children start to multiply 3digit x 3 digit and 4 digit x 2 digit … they should be confident with the abstract:

To get 744 children have solved 6 x 124

To get 2480 they have solved 20 x 124



Conceptual Variation, different ways to ask children to solve 6 x 23



With the counters, prove that

6 x 23 = 138

Explain why…

6x23= 23 x6

Mai had to swim 23 lengths, 6 times a week. How many lengths did she swim in one week?

Tom saved 23p three days a week. How much did he save in 2 weeks?

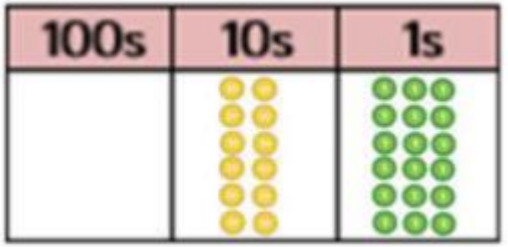
Find the product of 6 and 23.

$6 \times 23 =$

$\square = 6 \times 23$

$$\begin{array}{r} 6 \\ \times\ 23 \\ \hline \end{array} \qquad \begin{array}{r} 23 \\ \times\ 6 \\ \hline \end{array}$$

What is the calculation?
What is the product?



MENTAL CALCULATIONS FOR MULTIPLICATION

These are a **selection** of mental calculation strategies:

Doubling and halving

Applying the knowledge of doubles and halves to known facts. e.g. 8 x 4 is double 4 x 4

Using multiplication facts

Tables should be taught every day from Y1 onwards, either as part of the mental oral starter or other times as appropriate within the day. Pupils to be confident using the counting stick and able to recall and use multiplication and division facts.

Year 1	x2, x5 and x10 tables
Year 2	x2, x5, x10 and x3 tables
Year 3	x3, x6, x4 and x8 tables
Year 4	all tables up to 12 x 12

Using and applying division facts

Children should be able to utilise their tables knowledge to derive other facts. e.g. If I know 3 x 7 = 21, what else do I know?
30 x 7 = 210, 300 x 7 = 2100, 3000 x 7 = 21 000, 0.3 x 7 = 2.1 etc

Use closely related facts already known

13 x 11 = (13 x 10) + (13 x 1) = 130 + 13 = 143

Multiplying by 10 or 100

Knowing that the effect of multiplying by 10 is a shift in the digits one place to the left.

Knowing that the effect of multiplying by 100 is a shift in the digits two places to the left.

Partitioning

23 x 4 = (20 x 4) + (3 x 4) = 80 + 12 = 102

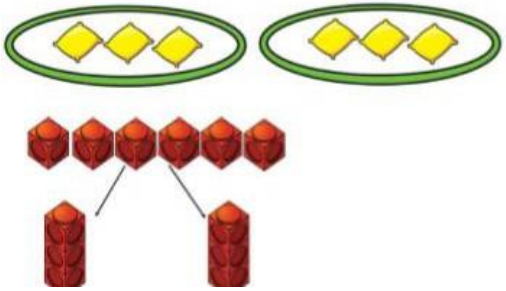
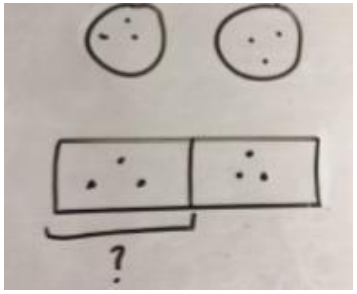
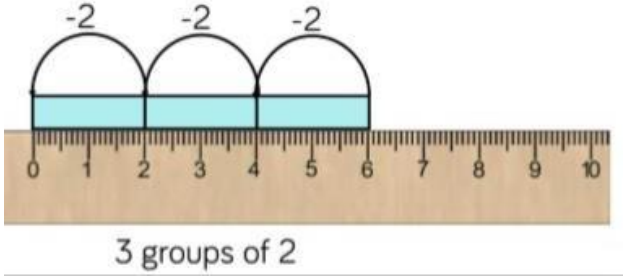
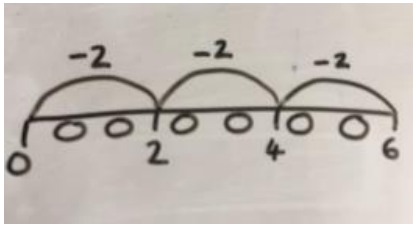
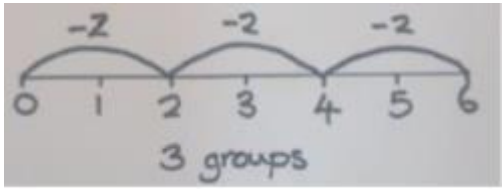
Use of factors

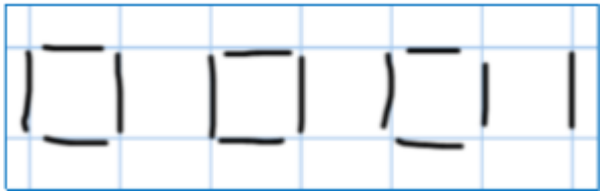
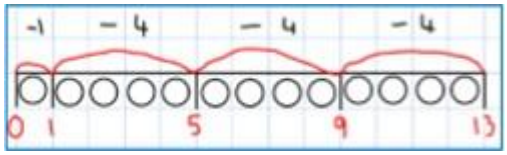
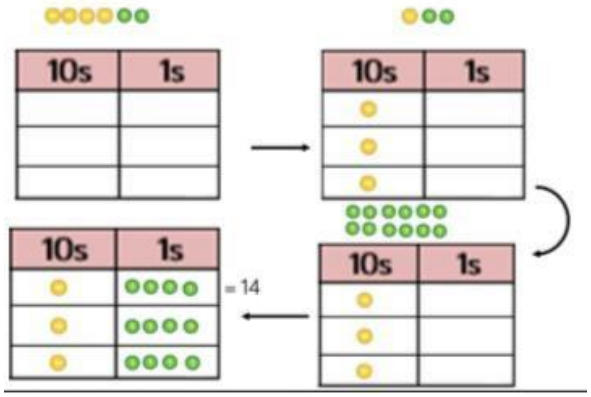
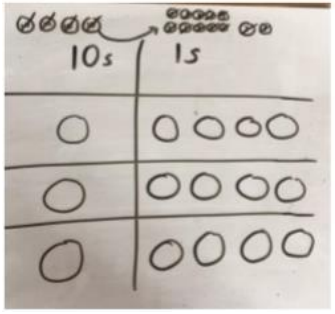
8 x 12 = 8 x 4 x 3

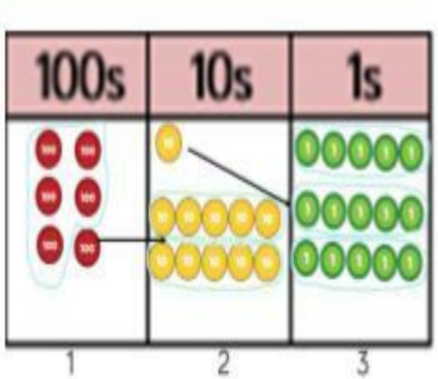
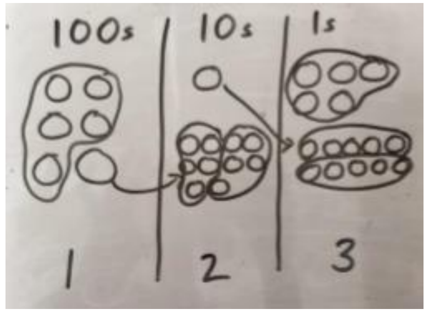
MANY MENTAL CALCULATION STRATEGIES WILL CONTINUE TO BE USED. THEY ARE NOT REPLACED BY WRITTEN METHODS.

CALCULATION POLICY FOR DIVISION

Key language which should be used: share, group, divide, divided by, half.

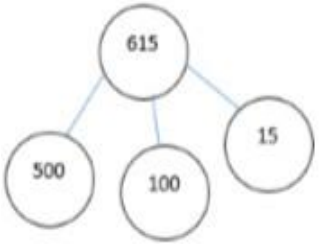
Concrete	Pictorial	Abstract
Sharing using a range of objects. 6 ÷ 2 	Represent the sharing pictorially. 	6 ÷ 2 = 3 
Repeated subtraction using Cuisenaire rods. 6 ÷ 2 	Children to represent repeated subtraction pictorially. 	Abstract number line to represent the equal groups that have been subtracted. 

2 digit ÷ 1 digit with remainders; using lollipop sticks or Cuisenaire rods. 13 ÷ 4 Using lollipop sticks to form wholes – squares are made because we are dividing by 4.  There are 3 whole squares, with 1 left over.	Children to represent the lollipop sticks pictorially.  There are 3 whole squares, with 1 left over. 	13 ÷ 4 = 3 remainder 1
Sharing using place value counters; 42 ÷ 3 = 14 	Children to represent the place value counters pictorially. 	42 ÷ 3 42 = 30 + 12 30 ÷ 3 = 10 12 ÷ 3 = 4 10 + 4 = 14

Short division (Bus Stop); with place value counters to group. 615 ÷ 5  1. Make 615 with place value counters. 2. How many groups of 5 hundreds can you make with 6 hundred counters? 3. Exchange 1 hundred for 10 tens. 4. How many groups of 5 tens can you make with 11 ten counters? 5. Exchange 1 ten for 10 ones. 6. How many groups of 5 ones can you make with 15 ones?	Children to represent the counters pictorially.  	Children to record the calculation using the short division scaffold. $\begin{array}{r} 123 \\ 5\overline{)6{}^{1}1{}^{1}5} \end{array}$

Conceptual Variation, different ways to ask children to solve 615 ÷ 5

Using the part whole model below, how can you divide 615 by 5 without using short division?



I have £615 and share it equally between 5 bank accounts. How much will be in each account?

615 pupils need to be put into 5 groups. How many will be in each group?

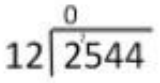
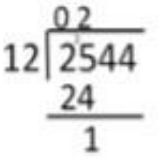
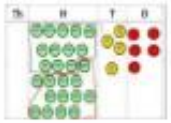
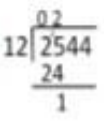
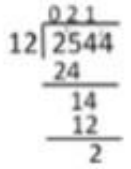
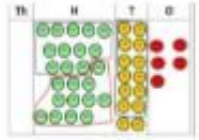
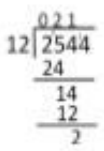
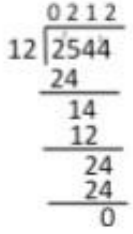
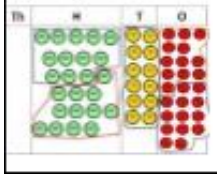
$5\overline{)615}$

615 ÷ 5 =

☐ = 615 ÷ 5

What is the calculation?

Long Division:

Concrete	Pictorial	Abstract
  2544 ÷ 12 How many groups of 12 thousands do we have? None	Children to represent the counters, pictorially and record the subtractions beneath.	 Step one- exchange 2 thousand for 20 hundreds so we now have 25 hundreds.
 Exchange 2 thousand for 20 hundreds.		 Step two- How many groups of 12 can I make with 25 hundreds? The 24 shows the hundreds we have grouped. The one is how many hundreds we have left.
  How many groups of 12 are in 25 hundreds? 2 groups. Circle them. We have grouped 24 hundreds so can take them off and we are left with one.		 Exchange the one hundred for 10 tens. How many groups of 12 can I make with 14 tens? The 14 shows how many tens I have, the 12 is how many I grouped and the 2 is how many tens I have left.
  Exchange the one hundred for ten tens so now we have 14 tens. How many groups of 12 are in 14? 1 remainder 2.		 Exchange the 2 tens for 20 ones. The 24 is how many ones I have grouped and the 0 is what I have left.
 Exchange the two tens for twenty ones so now we have 24 ones. How many groups of 12 are in 24? 2		

MENTAL CALCULATIONS FOR DIVISION

These are a **selection** of mental calculation strategies:

Doubling and halving

Knowing that halving is dividing by 2

Deriving and recalling division facts

Tables should be taught every day from Y1 onwards, either as part of the mental oral starter or other times as appropriate within the day. Pupils to be confident using the counting stick and able to recall and use multiplication and division facts.

Using and applying division facts

Children should be able to utilise their tables knowledge to derive other facts. e.g. If I know 3 x 7 = 21, what else do I know?

21 ÷ 3 = 7 21 ÷ 7 = 3 3 = 21 ÷ 7 and 7 = 21 ÷ 3

Dividing by 10 or 100

Knowing that the effect of dividing by 10 is a shift in the digits one place to the right.

Knowing that the effect of dividing by 100 is a shift in the digits two places to the right.

Use of factors

378 ÷ 21 378 ÷ 3 = 126 378 ÷ 21 = 18 126 ÷ 7 = 18

Use related facts

Given that 1.4 x 1.1 = 1.54 What is 1.54 ÷ 1.4, or 1.54 ÷ 1.1?